Quantitative Risk Assessment: A Pragmatic Approach

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Abstract

Risk has been defined and characterized qualitatively by many researchers, but quantitative assessments methods are rarely discussed in publication. Quantitative risk assessment lacks a concise approach and all research reviewed struggled to recommend definitive methods to measure this important concept.

In recent years, there has been a desire to adopt a quantitative methodological approach in decision-making, in addition to qualitative management approaches in various business disciplines. Decision-making processes, which rely largely on quantitative assessment, are becoming more computers automated. The algorithms for such automated decisions are written first in quantitative formats to model human decisions, increasing the appeal of quantitative risk assessment. This paper provides an example of a simple and pragmatic approach for quantitative measure of risk. It is consistent with the current ISO 31000 and the widely cited Kaplan and Garrick qualitative definitions. In addition, it leads to a much more vivid understanding of the term risk. The paper stipulates that a suitable methodology based on scientific concepts provides an acceptable tool in increasing understanding of risk assessment; prioritization and allocation of resources; and to facilitate the decision on comparative risk assessments.

Keywords

Risk, Expected Value, Mean, Variance

1. Introduction

There are a few qualitative definitions of risk (Carey & Burgkman, 2008; Kelman, 2003; Thywissen, 2006). In previous standards such as the AS/NZS 4360:2004, risk was defined as *a chance of an event occurring that has impact on objectives*. In the current standard, ISO 31000, risk has been defined by the International Standard Organisation (ISO) as the *effect of uncertainty on objectives*. (ISO 31000, 2009; Purdy, 2010). The difference is very subtle and there has not been any attempt to explain these definitions in quantitative terms.

Kaplan and Garrick, (1981), first defined risk quantitatively as a product of the probability of an event and the severity of its effect. This commonly cited quantitative definition has provided the foundation in many quasi quantitative methods, by multiplying the probability of occurrence of an impact and the magnitude. Sayers et al, (2002) later reaffirmed the definition as; "Risk is a product combination of the chance of a particular event, with the impact that the event would cause if it occurred". Hence, Risk, has two components – the chance (or probability) of the event occurring (which generates uncertainty), and the impact (or consequence) associated with that event.

The emphasis now in the global definition, ISO 31000, appears to have shifted from the *chance of occurrence* to the *uncertainty of the outcome*; with an effect that may be positive or negative. The unpredictable result is expressed relative to the conditions described in predetermined objectives. Whether the uncertainty is applied to the event or is associated with the end outcome will yield an *equivalent* (Epps, 2004) result because multiplication has a *communitative* property (borrowing the mathematical language and the meaning of equivalence here). That is to say, it does not matter which way randomness is seen and applied; the quantitative assessment of the result would be the same. Hence, there is no significant difference in the meanings of the earlier definition (AS/NZ/ISO 31000 2009) and the current ISO 31000. The two statements are similar except in the qualitative sense which has been discussed by others (Purdy, 2010).

Risk management, on the other hand, is outside the scope of this discussion. Risk assessment and management overlap with management prerogatives which are mandated by organisational policies. Risk management is commonly distinguished from risk assessment, even though some may use the term risk management to connote the entire process of risk assessment. In risk assessment, the assessors often attempt to answer the following set of triplet questions (Kaplan & Garrick, 1981).

- *What can go wrong?*
- *What is the likelihood that it would go wrong?*
- *What are the consequences?*

Answers to these questions helped them to identify, measure, quantify, and evaluated impacts. However, no discussion will be continued on risk management.

The existence of different qualitative definitions for risk (Kelman, 2003) is problematic for risk assessment and this has initiated this quantitative outlook. Although, there is no intention to make a judgment on any qualitative definition, all the versions purport a variation to predetermined objectives, and the fear of an unexpected result. This variation constitutes the risk in quantitative terms and forms the core issue in this paper. The dichotomy between uncertainty of result and the objectives presented by ISO 31000 (2009) brings out an issue of distinguishing the objectives, uncertainty and the result or outcome. Figure 1 exhibits these differences clearly and they are explained in detail subsequently.

According to ISO 31000 (2009), the chance of the occurrence of an impact is not as important as the risk result. One favorable argument for this assertion is that the likelihood of an impact could be high but the variation in the objective could be insignificant. Purdy (2010), described risk and stated that it is the *changing* of the magnitude and likelihood of consequences that result in the outcome. The consequence refers to the magnitude of the impact and likelihood is the probability of its occurrence; whether the effect is positive or negative, (benefit or loss). In quantitative terms, the *change in the result with the net effect is* a *variation.*

This paper distinguishes the variance as risk. The objectives of any project are also different from the expected result as risk is considered the random outcome. Risk occurs in an unpredictable manner which is characterized by uncertainty. Hence, objectives plus or minus risk equals result or outcome. (See Figure 1). This is the fundamental aspect of the ISO 31000 and it is consistent with other qualitative definitions such as Kaplan & Garrick, 1981. The quantitative way of thinking of the term risk is a description of the variance that occurs and affects the predetermined objectives.

If objectives are taken as a quantitative value then it is not difficult to measure the variance and make a decision on what the impact on the objectives would be. The unknown result can be described as a mathematical expectation which is a random variable of the sum of the likelihood or probability of each value multiplied by the magnitude.

The expectation is the average value or mean of a random variable or outcome. Therefore, project outcomes can be viewed as the values of a collection of independent, identically distributed random variables. The sample mean (or sample expectation) is defined as the expectation of the outcomes with respect to a distribution such as normal distribution shown in Figure 1.

2. The Risk Rationale: ISO 31000

Risk is about the expectation of predetermined objectives which is characterised by a value and a good understanding of quantitative risk assessment begins with quantifying value. Majority of professionals and managers in industry and government emphasises on value and have devoted a large portion of their time and resources to the task of improving their understanding of it but only in qualitative terms.
Value is an all-inclusive term which models all aspects of criteria in the objectives such as financial, quality, health & safety, environmental issues etc. In quantitative terms, it is represented by the mean of these discrete variables; such as value, risk and the result or outcome. Using the normal arithmetic mean, this implies that value is quantified as the arithmetic mean or average.

For example, it is well known that the average of the set of results such as [1, 2, 3, 4, and 5] is 3, assuming that these numbers are distinguished project outcomes. The average is simply obtained by just adding up 1+2+3+4+5 = 15, and dividing the sum by the quantity 5; which gives the value of 3, the objective. [1, 2, 4 and 5} are outcomes which have been impacted by risks. The risk values are [-2, -1, +1 and +2]. If the outcomes have been [3, 3, 3 and 3], risk would have been zero and all results or project outcomes have been successful. Project outcomes can be viewed as the values of a collection of independent identically distributed random variables. The mean (or sample expectation) is defined as the expectation of the values with respect to the empirical distribution for the outcomes. This makes it simply the arithmetic average of the values.

The mathematical expectation of an objective is defined by the expected value, *E(x).* This expectation is given by the formula:

$$E(x) = \sum sxp(x) \qquad \textbf{Equation 1}$$

where *x* is the objective or value and *p(x)* is the likelihood of its occurrence. *s* is the number of possible outcomes or sample space of the results/outcomes. In the above example, *s* = 5. For a single project *s* = 1.

Hence, the commonly cited quantitative expression for risk. (a single situation, *s* = 1):

$$Risk = Consequence\ x\ Likelihood\ of\ occurrence \qquad \textbf{Equation 2}$$

$$E(x) = \sum xp(x) \qquad \textbf{Equation 3}$$

Equation 2 and 3 are the same for a single project. However, what eludes managers is, risk is an expected value and a variance, a proportion of the objectives as shown in Figure 1. The expectation is the average value or mean of the all possible results within a sample space *s*.

Value provides the leverage in the analysis of risk assessment and decision-making, and the approach to developing criteria on which to base a decision (National Risk Council, 1996).
The difference between Eq. (2) and (3) is that the latter has a value that is precise. It is controlled largely by uncertainty which in turn affects the impact on an objective.
Purdy (2010) added that the new standard supports a new, simple way of thinking about risk and is intended to begin the process of resolving the many inconsistencies and ambiguities that exist between many different approaches and definitions. Although, most decision makers accept the new standard, it does create challenges for those who use language and approaches that are unique to their area of work, but different from the new standard and guide.
A scientific explanation for the change is the ultimate path towards the standardization.

Figure 1 which is shown below illustrates clearly the quantitative outlook of the ISO 31000 definition of risk:

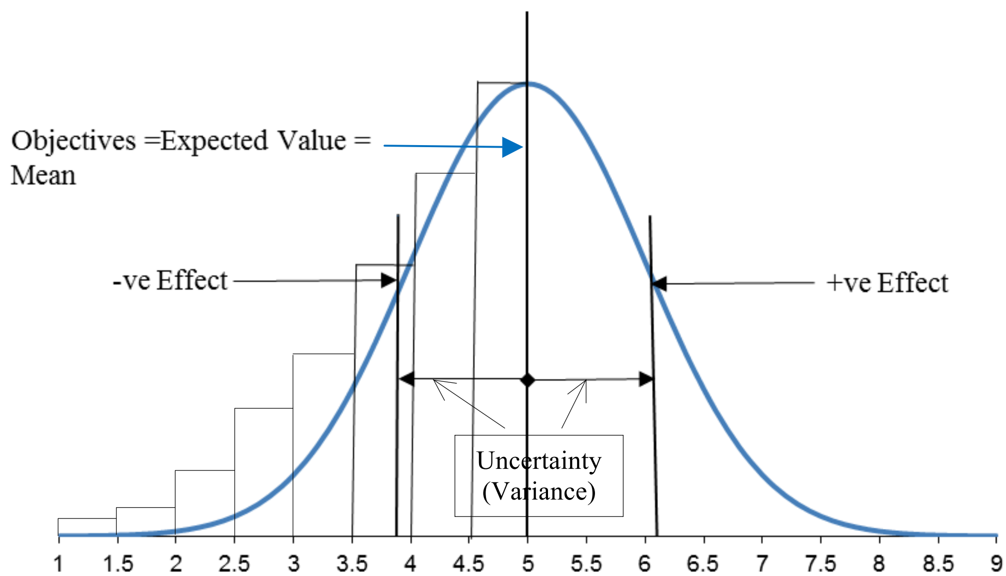

Figure 1: "*Effect of Uncertainty on objectives*" ISO 31000

Hence, the ISO global definition delineates risk simply as *the variance of an expected value*.

The variance is the risk and it is the parameter which is susceptible to uncertainty. This definition is consistent with the definitions of Kaplan and Garrick, (1981) and ISO 31000. It is also consistent with other qualitative definitions ((Haimes et al., 2002; Johnson-Payton, Haimes, & Lambert, 1999; Karlsson & Haimes, 1989). The only issue is that qualitative research does not recognise that risk implies variance from an expected value. Commonly, the variance is negative when it is considered as a loss. A Civil Contractor may quote risk as $250,000 per day, meaning it would cost $250,000 per day in case of time overrun. Hence, this paper proposes the use of a *standard deviation* and the *coefficient of variation* as one way of measuring risks and to provide a common denominator to compare different risks when dealing with different situations or scenarios. It eliminates local variables and provides a bench mark to make decisions when comparing risks. It is dimensionless and does not depend on any unit or currency. Also known as Scaled Risk, it is determined as:

$$Scaled\ \ Risk\ \ (Coefficient\ \ of\ \ Variation) = \frac{\sigma}{E(x)}$$ **Equation 4**

$$Risk \quad = \qquad \sigma = \sqrt{\sum_{i=1}^{N}\left(X_i - E\left(X\right)\right)^2 P\left(X_i\right)}$$ **Equation 5**

$E(x_i)$ = expected value of objectives,

σ = variance or standard deviation,

$P(x_i)$ = likelihood of occurrence

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The coefficient of the variation is a very useful tool that enables the manager to: (1) compare one expected value with other expected values; and, (2) make the decision as to which impact will be designed for resilience. Further, the tool can also be used to compare one impact with other impacts elsewhere, even when the data set or return periods differ and take into account potential solutions for risk reduction.

Example of the Pragmatic Approach.

Preamble

In this example, an impact risk assessment of road infrastructure is quantitatively carried out. The methodology is based on the approach outlined above. Using data which represented yearly mean magnitudes of impacts, the expected values and risks have been determined to show the application of the concept. The impacts of climate change are values measured by Thornthwaite Moisture Index, (TMI). The example goes on further to show how the risks at different locations were compared.

Road infrastructure can be affected by climate change through excessive rainfall, and temperature, which cause deterioration of the pavement. Thornthwaite Moisture index is a climatic moisture index used extensively by road researchers, (Thornthwaite, 1948), and others as an indicator of the supply of water in an area relative to the demand under prevailing climatic conditions. Thornthwaite, (1948), originally introduced the concept of "Potential Evaporation" and developed an empirical formula defining the moisture index (I_m).

Quantifying the Expected Value (Impact)

Given by Equation 6, the TMI defines the total amount of water that would evaporate and transpire if it is always available for use. It is a dimensionless ratio which is expressed as:

$$(\boldsymbol{TMI}),\ \boldsymbol{I_m} = \frac{\boldsymbol{s} - 0.6\boldsymbol{d}}{\boldsymbol{n}} \ \boldsymbol{x}\ 100 \qquad \textbf{Equation 6}$$

s = impact moisture in the soil

d = deficient moisture in the soil, and

n = necessary quantity of moisture for plants.

TMI may be used to provide an equivalent measure of climate change because it expresses the index as a function of temperature and precipitation and the occurrence of extreme events which are random and cause damage to road pavements.

The Quantitative Risk Assessment.

The technique for estimating the impact risk follows the procedure above. The values used here are yearly averages. TMI models weather changes and complies with the Expert Team on Climate Change Detection and Indices (ETCCDI) requirements (CCl/CLIVAR/JCOMM, 2009; Karl, Nicholls, & Ghazi, 1999). However, it is cautioned here that the example shown here is for the purpose of illustrating quantitative risk assessment. The data may not necessary be considered suitable for climate change impact assessment debate.

The road TMIs shown in Table 1 & 2 have been used to calculate longer run (future) mean for the expected impact which is denoted by the symbol $E(X_i)$ and shows the impact risks associated with the values over a threshold value of 100, column **(c).** The threshold paradigm implies that some climate change can be tolerated as normal. For effects for which a threshold value can be identified, this is done in a quantitative way by comparison of estimates of actual levels of weather extremes with observed adverse levels which are unacceptable for humane standards. The expected impact is defined as the weighted average of measured numerical TMI values of X_i, with the respective frequencies or probabilities used for the weights, column **(g).** Table 1, shows the analysis at the location named as QL02, and Table 2 is for a different location at QL04, from 1960 to 2007. The aim of the analysis shown here is to work out the risks, rank and determine which location is hit hardest by climate change.

Table 1: Analysis of Impact Risk at QL02

Year **(a)**	Index Value **(b)**	Impact Value02 (I-100) **(c)**	Frequency **(d)**	Probability, $P(X_i)$ **(e)**	Expected Impact, $E(X_i)$ **(f)**	Weighted Square **(g)**
1960	19	81	1.00	0.0833	6.7333	240.2329
1961	61	39	1.00	0.0833	3.2333	11.3913
1962	76	24	1.00	0.0833	2.0000	0.8051
1963	100	0	1.00	0.0833	0.0000	61.2385
1964	73	27	1.00	0.0833	2.2833	0.0071
1965	100	0	1.00	0.0833	0.0000	61.2385
1966	54	46	1.00	0.0833	3.8333	29.7413
1967	100	0	1.00	0.0833	0.0000	61.2385
1968	81	19	1.00	0.0833	1.5667	5.7524
1969	53	47	1.00	0.0833	3.9083	32.6425
1970	57	43	1.00	0.0833	3.5500	19.9993
1971	100	0	1.00	0.0833	0.0000	61.2385
			12.00		27	586
Expected Impact		27				
Highest Impact		81				
Lowest Impact		0	Period		Expected Impact	Risk
Total No of Impact		12	1960	1971	27	24
Expected Impact Risk		24	(i)		Scaled Risk	89%

Table 2 Analysis of Impact Risk at QL04

Year **(a)**	Index Value **(b)**	Impact Value04 (I-100) **(c)**	Frequency **(d)**	Probability $P(X_i)$ **(e)**	Expected Impact E (X_i) **(f)**	Weighted Square **(g)**
1972	100	0	1.00	0.0833	0.0000	73.7552
1973	100	0	1.00	0.0833	0.0000	73.7552
1974	100	0	1.00	0.0833	0.0000	73.7552
1975	40	60	1.00	0.0833	5.0083	76.7602
1976	100	0	1.00	0.0833	0.0000	73.7552
1977	17	83	1.00	0.0833	6.9417	238.9669
1978	83	17	1.00	0.0833	1.4167	13.5469
1979	11	52	1.00	0.0833	4.3333	41.2552
1980	41	59	1.00	0.0833	4.9417	72.7669
1981	66	34	1.00	0.0833	2.8250	1.4352
1982	49	51	1.00	0.0833	4.2833	39.0602
1983	100	0	1.00	0.0833	0.0000	73.7552
			12.00		30	853
Expected Impact		30				
Highest Impact		83				
Lowest Impact		0	Period		Expected Impact	Risk
Total No of Impact		12	1972	1983	30	29
Expected Impact Risk		30	(ii)		Scaled Risk	98%

3. Discussion

Quantitative risk assessment is a methodology used to organise and analyse scientific information to estimate probability and severity of an adverse event. The approach shown above incorporates uncertainty automatically in the risk estimate. The probabilities are calculated based on their relative frequencies in a group. All the impact values, (Table 1, column c), occurred once therefore their frequencies are one and it is not difficult to divide by the total number in the period of study, (column a), to obtain the probabilities (column e). This shows, *inter alia*, that the risk manager does not need to have a previous knowledge of the occurrence of an impact to determine the likelihood of occurrence in the future. All unpredictable results will occur once for the first time. Thereafter, its frequency will change and so its likelihood of occurrence will improve.

The climate change data have been selected to illustrate the methodology only. The analysis is not meant for climate change impact debate. Column (c) shows how the risk values are determined using the expected impact values. Processing the impacts further provides the expected impact risks and the relative (scaled) risks. The main issues are the *expected impacts value calculation*, *the variance* and *the covariance* in accordance with Eq. (4) & (5) in order to show the application.

This example begins with assumed quantitative data, which are shown in column **(b).** These are climate change impact values or TMI values as measures of climate change impact. The change is determined over a threshold value of 100, column **(c).** The expected value is obtained by summing the possible values times their relative occurrence (probabilities) in each group. To get the standard deviation, the average square deviation from the expected value, column **(g),** is first calculated, and then the square root is taken.

From Table 2, it can be seen that the location identified as QLD 04 has the highest risk, i.e. 98% compared with the location QLD 02, which has risk of 89% and their corresponding expected impact risks are 30 and 24 respectively. Hence, the impact is worse at the QLD 04 location compared with QLD 02.

4. Conclusion

The ISO 31000 is an all-in-one definition which has been illustrated quantitatively and explained comprehensively in this paper. A corollary of this global definition provides the necessary bridge between qualitative and quantitative understanding. This would enable the manager to formulate a scientific methodological procedure for risk assessment.

Quantitative risk assessment may be difficult but utilises methodology based on scientific concepts and provides an acceptable tool and increases understanding of risk assessment. Arguably, it is the most important step in the risk management process, although it may also be the most difficult and prone to error. However, once value has been quantified, the steps to properly deal with risk are much more programmatic.

This paper has laid out a holistic framework that can facilitate high level assessment of risks inherent in a set of complex systems. The expected value approach incorporating likelihood of occurrence is the coherent credible approach in dealing with uncertainty in quantitative risk assessment. This is the subtle issue about risk which makes quantitative assessment appear difficult and elusive with uncertainty decomposed into variability and incertitude.

It has also expanded our knowledge of the meaning of risk, objectives, uncertainty, and result or outcome with ISO 31000; especially within qualitative and quantitative contexts and may stimulate further quantitative appraisal and debate within the research community. Risk managers may organise and analyse qualitative information to estimate the probability and severity of an adverse event and rank effects at different locations. It may be found very useful in the area of public health and in making environmental decisions where funding is often made in the light of competing demand for limited resources.

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